Unit 1 - Limits and Rates of Change

- Given a function *f*, the limit of *f*(*x*) as *x* approaches *c* is a real number *R* if *f*(*x*) can be made arbitrarily close to *R* by taking *x* sufficiently close to *c* (but not equal to *c*). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = R$.
- The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
- A limit might not exist for some functions at particular values of *x*. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
- Numerical and graphical information can be used to estimate limits.
- Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
- The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
- A function *f* is continuous at *x* = *c* provided that *f*(*c*) exists, $\lim_{x \to c} f(x)$exists, and $\lim_{x \to c} f(x) = f(c)$.
- Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
- Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
- Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.
- The difference quotients $\frac{f(a+h)-f(a)}{h}$and $\frac{f(x)-f(a)}{x-a}$express the average rate of change of a function over an interval.
- The instantaneous rate of change of a function at a point can be expressed by $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$or $\lim_{h \to 0} \frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted *f*′(*a*).

Unit 2 - Derivatives

- The difference quotients $\frac{f(a+h)-f(a)}{h}$and $\frac{f(x)-f(a)}{x-a}$express the average rate of change of a function over an interval.
- The instantaneous rate of change of a function at a point can be expressed by $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$or $\lim_{h \to 0} \frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted *f*′(*a*).
- The derivative of *f* is the function whose value at *x* is $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$provided that this limit exists.
- For *y* = *f*(*x*), notations for the derivative include $\frac{dx}{dy}$, *f*′(*x*), and *y*′.

- The derivative can be represented graphically, numerically, analytically, and verbally.
- The derivative at a point can be estimated from information given in tables or graphs.
- Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
- The chain rule provides a way to differentiate composite functions.
- The chain rule is the basis for implicit differentiation.
- Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f.
- Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y''. Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$,or $f^{(n)}(x)$.
- A continuous function may fail to be differentiable at a point in its domain.
- If a function is differentiable at a point, then it is continuous at that point.
- The unit for $f'(x)$ is the unit for f divided by the unit for x.
- The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
- The derivative at a point is the slope of the line tangent to a graph at that point on the graph.
- The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
- The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
- The derivative can be used to solve related rate problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
- The derivative can be used to express information about rates of change in applied contexts.

Unit 3 - Applications of Derivatives

- Asymptotic and unbounded behavior of functions can be explained and described using limits.
- First and second derivatives can provide information about the function and its graph including intervals of increase or decrease, local and global extrema, intervals of upward or downward concavity, and points of inflection.

- Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
- Key features of the graphs of *f*, *f’*, and *f’’* are related to one another.
- The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
- If a function *f* is continuous over the interval [*a, b*], and differentiable over the interval (*a, b*), the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.

Unit 4 - Integrals

- The antiderivative of a function *f* is a function *g* whose derivative is *f*.
- Differentiation rules provide the foundation for finding antiderivatives.
- A Riemann sum, which requires a partition of an interval *I*, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
- The definite integral of a function *f* over the interval [a, b], denoted by $\int_a^b f(x)dx = \lim_{max\Delta x_1 \to 0} \sum_{i=1}^{n} f(x_i)\Delta x_i$ where x_i is a value in the *i*th subinterval, Δx_i is the width of the *i*th subinterval, *n* is the number of subintervals, and $max\Delta x_i$ is the width of the largest subinterval. Another form of the definition is $\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$ where $\Delta x_i = \frac{b-a}{n}$ and x_i is a value in the *i*th subinterval.
- The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
- Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, or verbally.
- Definite integrals can be approximated as a left Riemann sum, right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
- In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
- Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
- The definition of the definite integral may be extended to functions with removable or jump discontinuities.
- If *f* is a continuous function on the interval [a, b], then $\frac{d}{dx}(\int_a^x f(t)dt) = f(x)$ where *x* is between *a* and *b*.

- Graphical, numerical, analytical, and verbal representations of a function *f* provide information about the function *g* defined as $g(x) = \int_a^x f(t)dt$.
- The function defined by $F(x) = \int_a^x f(t)dt$ is an antiderivative of *f*.
- If *f* is continuous on the interval [a, b] and *F* is an antiderivative of *f*, then $\int_a^b f(x)dx = F(b) - F(a)$.
- The notation $\int f(x)dx = F(x) + C$ means that $F'(x) = f(x)$, and $\int f(x)dx$ is called an indefinite integral of the function *f*.
- Many functions do not have closed form antiderivatives.
- Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables. (DO NOT INCLUDE integration by parts or nonrepeating linear partial fractions).
- A function defined as an integral represents an accumulation of a rate of change.
- The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
- The limit of an approximating Riemann sum can be interpreted as a definite integral.

Unit 5 - Applications of Integration

- Areas of certain regions in the plane can be calculated with definite integrals.
- Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
- The average value of a function *f* over an interval [a, b] is $\frac{1}{a-b}\int_a^b f(x)dx$.

Unit 6 - Differential Equations

- Slope fields provide visual clues to the behavior of solutions to first order differential equations.

Unit 7 - Inverse Functions

- Limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hopital's Rule.
- Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.